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SINGULAR PERTURBATION ANALYSIS OF AOTV RELATED TRAJECTORY OPTIMIZATION PROBLEMS

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SUMMARY

Research during this period has concentrated on the problem of aeroassisted orbital plane change. This maneuver requires the use of three impulses - one to deorbit, one to reorbit and one to recircularize at the new orbit. The orbit plane change is effected entirely in the atmosphere through the use of lift and bank angle control. For circular orbits of nearly equal radii, it can be shown that the fuel consumption is minimized by minimizing the energy loss in the atmospheric portion of the trajectory. The research explores the use of singular perturbation theory to develop an optimal guidance law for the atmospheric portion.

The results to date indicate that singular perturbation methods can be applied; however, a difficult terminal boundary analysis is required. The reduced solution models only the heading rate dynamics, and produces a realistic profile (altitude versus energy) and control to be flown. A large terminal boundary layer is required to match the terminal constraint on altitude. Most of our effort has been directed at approximate methods for solving the terminal boundary layer equations. The equations result from an analysis of altitude and flight path angle dynamics on the same time scale. A nonlinear control law was derived which produces near optimal results. However, the current solution is difficult to implement because it requires two switches in the control solution that are heading and altitude dependent. In general, the solution is very sensitive to switching times. We propose two alternatives to be investigated during the next reporting period. The first relies on a linearization of the necessary conditions about the reduced solution and the second will examine the analysis of altitude and flight path angle dynamics on separate boundary layers.

1. PROBLEM FORMULATION

The following three state model has been the subject of our current research

$$\dot{\psi} = C_L^* \rho S V \lambda \sin\mu / 2m\cos\gamma \tag{1}$$

$$\varepsilon\dot{h} = V\sin\gamma \tag{2}$$

$$\varepsilon\dot{\gamma} = C_L^* \rho S V(\lambda\cos\mu + M\cos\gamma)/2m \tag{3}$$

where

$$M(h,V) = (2m/C_L^* S)[1-\bar{\mu}/V^2 r]/\rho r \tag{4}$$

$$r = r_s + h \tag{5}$$

and $\bar{\mu}$ is the gravitational constant. The objective is to minimize the energy loss

$$J = -\int_0^{t_f} \dot{E}\, dt \tag{6}$$

where E is the total energy per unit mass

$$E = V^2/2 - \bar{\mu}/r < 0 \tag{7}$$

The expression for the energy rate in (6) is

$$\dot{E} = -C_D^* (1 + \lambda^2)\rho S V^3/4m \tag{8}$$

where a parabolic drag polar form is used to define the drag coefficient

$$C_D = C_{Do} + KC_L^2 \tag{9}$$

In the above equations the superscript * denotes the lift and drag coefficient values at maximum L/D

$$C_L^* = (C_{Do}/K)^{1/2} \qquad C_D^* = 2C_{Do} \tag{10}$$

The controls are bank angle (μ) and the normalized lift coefficient

$$\lambda = C_L/C_L^* \tag{11}$$

Note that in this formulation we treat E as constant, but account for the energy loss through the performance index.

In [1] the sensible atmosphere is assumed to occur at h_o = 200,000 ft. The starting velocity and flight path angle (V_o,γ_o) are derived using a deorbit impulse ΔV_1 from circular orbit at h_c = 100 nm, which is optimized for the atmospheric maneuver of interest. The initial heading angle is taken as zero. In the SPT formulation, altitude appears as a control variable in the reduced problem. The optimal solution has the form

$$h^* = h(E) \tag{12}$$

For comparison purposes, in this study the starting energy is chosen to match that of [1], and h_o, V_o are derived from (7) and (12). From conservation of energy this results in the same deorbit impulse, but slightly different values for h_o, V_o. The initial flight path angle is derived from conservation of angular momentum.

$$\gamma_o = -\cos^{-1}[(r_s + h_c)(V_c - \Delta V_1)/(r_s + h_o)V] \tag{13}$$

where r_s is the mean earth radius and $V_c = [\bar{\mu}/(r_s+h)]^{1/2}$. The vehicle begins the maneuver with a mass m_c and, as a result of the deorbit impulse, the mass for the atmospheric portion is given by

$$m = m_c \exp(-\Delta V_1/C) \tag{14}$$

where C is the characteristic velocity. The terminal conditions are:

$$h(t_f) = 200{,}000 \text{ ft}, \qquad \psi(t_f) = \psi_f > 0 \tag{15}$$

Since the condition on $h(t_f)$ is lost in the reduced solution (12), a terminal boundary layer correction is required.

2. SINGULAR PERTURBATION ANALYSIS

2.1 Reduced Problem

Setting $\varepsilon = 0$ in (1-3) the necessary conditions for optimality become

$$H_o = \lambda_\psi \dot{\psi} - \dot{E} = 0 \tag{16}$$

$$\gamma = 0 \qquad\qquad \lambda\cos\mu = -M \tag{17}$$

$$\mu_o,\ h_o = \arg\min_{h,\mu} \{\dot{\psi}/\dot{E}\} \tag{18}$$

It can be shown that this results in the following reduced solution:

$$\lambda_o = (1 + 2M_o^2)^{1/2} \tag{19}$$

$$\sin\mu_o = [(1 + M_o^2)/(1 + 2M_o^2)]^{1/2} \tag{20}$$

$$h_o = \arg \min_h \{V^2(1 + M^2)^{1/2}\}|_{E = \text{const.}} \tag{21}$$

where M_o is the value of M for $h = h_o$. The quadrant for the bank angle in (20) is resolved based on the following inequalities:

$$0 < \mu_o < \pi/2 \text{ for } M < 0 \tag{22}$$

$$\pi/2 < \mu_o < \pi \text{ for } M > 0 \tag{23}$$

It can be seen from the above solution that M plays a crucial role in the solution process. In [1], M was treated as a constant in the dynamics.

Since most of the energy is kinetic, V is weakly dependent on h for constant E. This can readily be seen from (7) and (5) where changes in h give rise to small changes in r. Thus, the minimization in (21) results in a value for M very close to zero. The interpretation is that the maneuver should be performed at an altitude where gravitational and centripetal forces nearly cancel one another. For M small, it can be seen from (19,20) that the maneuver is performed at near maximum L/D and at near 90 of bank angle. These results are in good agreement with the results in [1]. Figure 1 compares the altitude profiles derived from (21) with the true optimal profile taken from [1]. The need for a terminal boundary layer analysis is evident in this figure. However, if the vehicle was not required to exit the atmosphere, the reduced solution may be sufficiently accurate.

2.2 Boundary Layer Problem

A boundary layer analysis is required to obtain a guidance law that will both follow the altitude profile defined by (21) (initial boundary layer) and

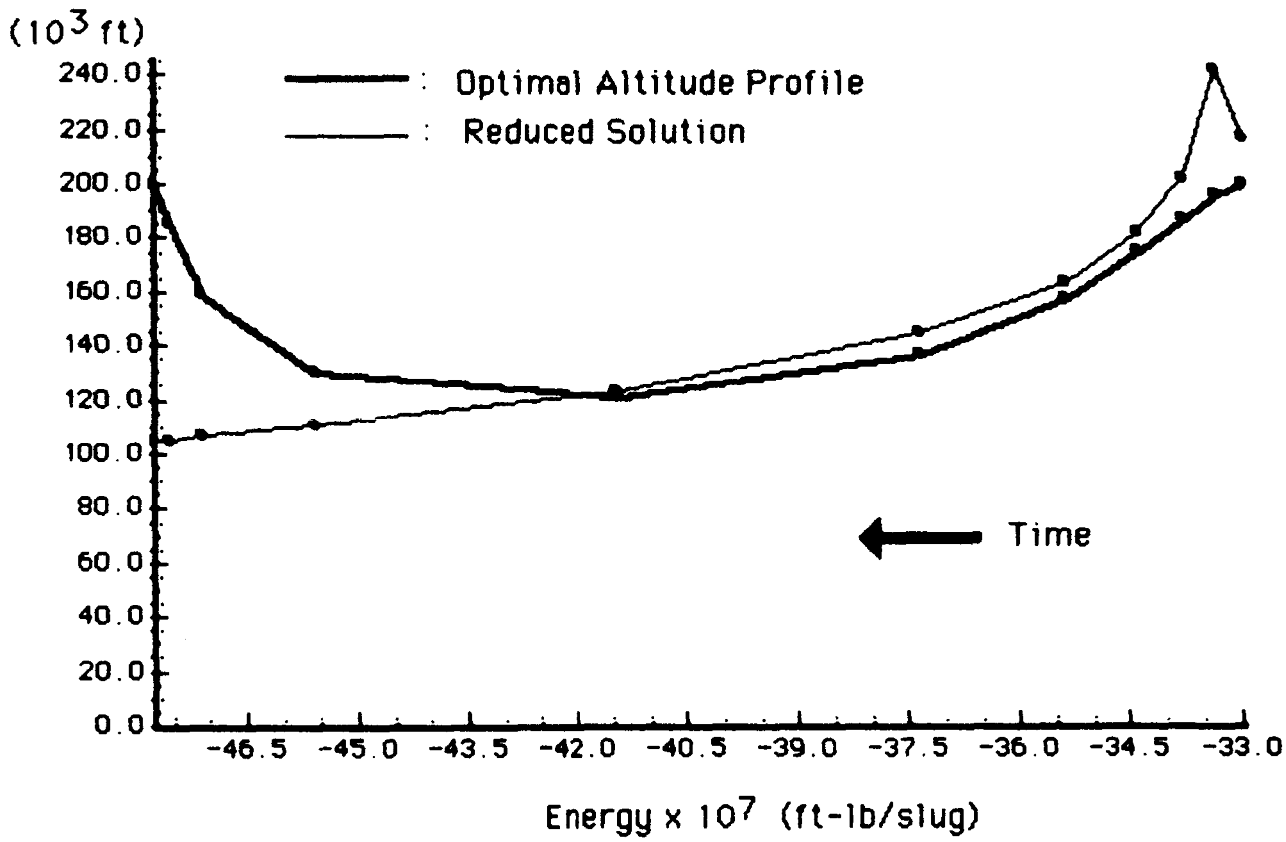

Figure 1. Comparison of the reduced solution with the true optimal profile.

satisfy the terminal constraint on altitude (terminal boundary layer). The necessary conditions in the boundary layer are:

$$H_{BL} = \lambda_\psi^o \dot{\psi} + \lambda_h V \sin\gamma + \lambda_\gamma \dot{\gamma} - \dot{E} = 0 \tag{24}$$

$$\partial H_{BL}/\partial L_1 = 0, \qquad \partial H_{BL}/\partial L_2 = 0 \tag{25}$$

where λ_ψ^o is determined in the reduced solution from (16)

$$\lambda_\psi^o = \dot{E}^o/\dot{\psi}^o \tag{26}$$

using the solutions for λ_o, μ_o and h_o. In (25), L_1 and L_2 represent the horizontal and vertical components of lift coefficient

$$L_1 = \lambda \sin\mu \qquad L_2 = \lambda \cos\mu \tag{27}$$

which are now used as control variables in place of λ and μ.

The first condition in (25) results in

$$L_1^* = (V_o/V)^2 (1 + M_o)^{1/2}/\cos\gamma \tag{28}$$

where M_o, V_o are the values of M and V corresponding to $h = h_o$ for the current value of E. This solution approaches the corresponding reduced solution as h approaches h_o.

The second condition in (25) yields

$$L_2^* = -(C_L^*/C_D^* V^2)\lambda_\gamma \tag{29}$$

which can also be shown to approach the reduced solution as h approaches h_o, where

$$\lambda_\gamma^o = C_D^* V_o^2 M_o / C_L^* \tag{30}$$

Unfortunately, evaluation of λ_γ needed in (29) requires the solution of a two-point boundary value problem. When close to the reduced solution it may be possible to use (30), which results in the following expression for flight path angle rate

$$\dot{\gamma} = C_L^* \rho SV(M\cos\gamma - V_o^2 M_o / V^2)/2m \tag{31}$$

For γ near zero and h near h_o, (31) simplifies to

$$\dot{\gamma} = C_L^* \rho SV_o(M - M_o) \tag{32}$$

To obtain a feedback solution for the general case we neglected the second term in (24). This was done on the basis that $\lambda_h^o = 0$ and γ is small over the entire optimal trajectory. This results in the following explicit solution for L_2

$$L_2^* = - M\cos\gamma \pm (M^2\cos^2\gamma - L_1^2 + 1)^{1/2} \tag{33}$$

The first term on the right hand side of (33) is simply the lift required to maintain zero flight path angle rate. The second term is always > 0 and asymptotically approaches zero as $h \rightarrow h_o$ and $\gamma \rightarrow o$. Thus this solution also asymptotically approaches the reduced solution. Both solutions in (33) satisfy the conditions that H_{BL} is minimized and $H_{BL} = 0$. During the initial boundary layer the + sign is used when $h < h_o$ to generate a positive flight path angle rate, and the minus sign used when $h > h_o$. The corresponding value of the costate variable is

$$\lambda_\gamma^* = (\lambda_\psi^o \, \dot{\psi} - \dot{E}^*)/\dot{\gamma}^* \tag{34}$$

which approaches an indeterminate form (0/0) as h h_o and γ 0. The + sign is used to initiate the terminal boundary layer.

At this time repeated trial runs are required to determine the switching time so that the desired final heading is achieved when the altitude reaches 200,000 ft. Also, a characteristic of these profiles is that L_1 remains close to 1.0 throughout, while M grows to a large negative number near the end (on the order of -2.0). This is due to the presence of ρ in the denominator of (4). Thus there is every indication that the sign should be switched again in (33) prior to the end of the trajectory so that L_2 again becomes small. This is also a general characteristic of the optimal profiles in [1].

From (29) it is apparent that L_2 should be a continuous function of time. There is a discontinuity that occurs at the switch to the terminal boundary layer which is a consequence of the singular perturbation approximation. A second discontinuity occurs at the second switch which is a consequence of neglecting the second term in (24). However, it was observed that the second term in (33) passes through a minimum during the ascent phase, and the second switch was executed at that time to minimize the discontinuity. It is felt that this should more closely approximate the true solution if we were able to retain the second term in (24) in the analysis, and still preserve an explicit solution for L_2.

A comparison of the resulting flight path with that in [1] for a 40 plane change is illustrated in Figure 2. Table 1 compares the impulses required for the maneuver. Note that the singular perturbation solution

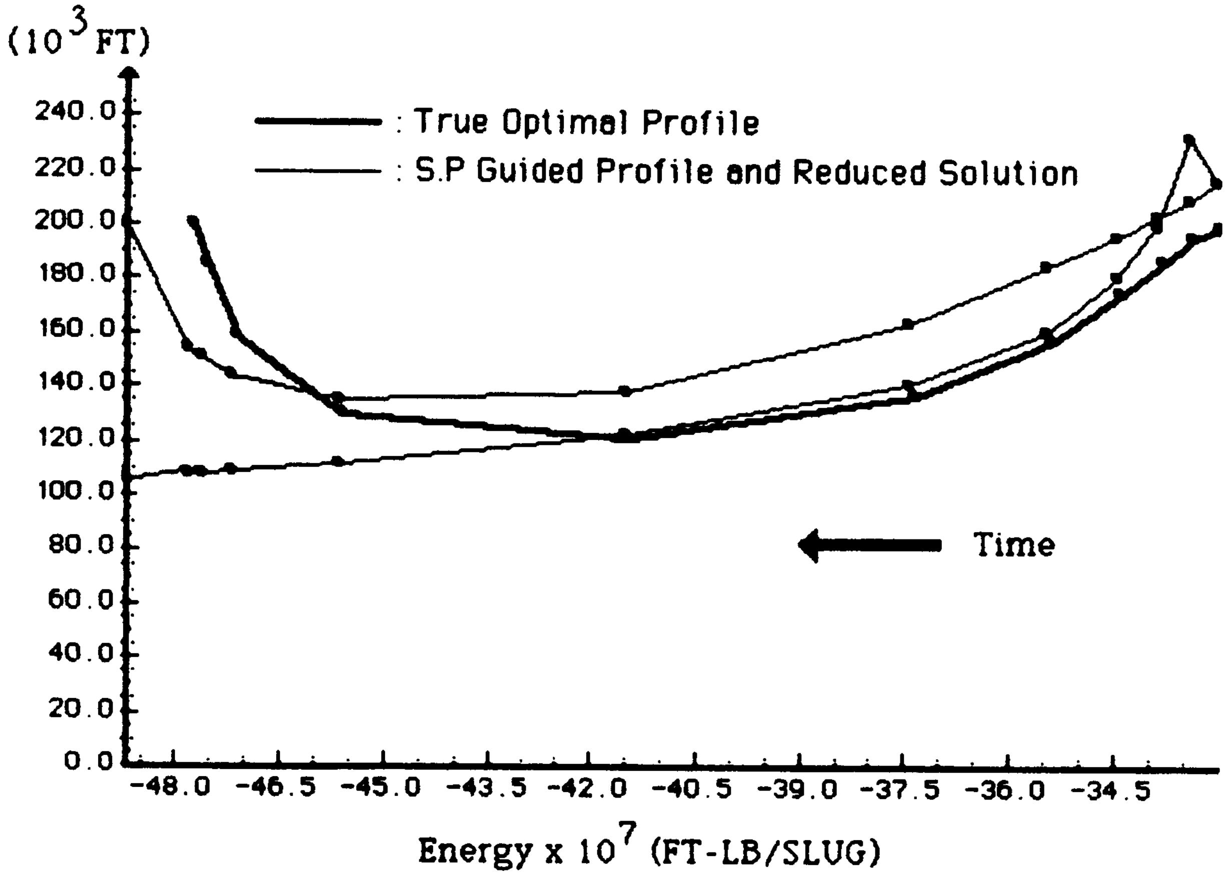

Figure 2. Comparison of the guided S.P. solution with the reduced solution and the true optimal profile.

TABLE 1

COMPARISON OF TOTAL IMPULSE AND FUEL FRACTION
REQUIRED FOR A 40^0 PLANE CHANGE MANEUVER

GUIDANCE LAWS	DEORBIT IMPULSE (ft/s)	BOOST IMPULSE (ft/s)	REORBIT IMPULSE (ft/s)	TOTAL IMPULSE (ft/s)	FUEL FRACTION
OPTIMAL	125.	6470.	177.	6772.	.49
S.P. SOLUTION	126.	6642.	214.	6982.	.50
GUIDED SOLUTION	374.	7651.	122.	8147.	.56
SINGLE IMPULSE	*	*	*	17497.	.83

results in a fuel fraction close to the true optimal solution, and is considerably better than the guided solution in [1]. A comparison to the fuel fraction needed for a purely impulsive maneuver is also given which clearly demonstrates the advantage of aero-assisted orbital transfer.

3. FUTURE WORK

During the next reporting period we plan to investigate two alternatives to constructing a boundary layer solution. The first is based on a linearization of the necessary conditions in the boundary layer to obtain a linear feedback solution without neglecting the second term in (24) This method has been previously used in [2]. The second approach analyzes the altitude and flight path angle dynamics in separate layers [3]. This approach also will yield a feedback solution form, but one which is nonlinear.

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